Topological Origin of Black Hole Entropy: A Field-Theoretic Model Based on Chromocharge Field Collapse and Bose-Einstein Condensation  
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Abstract:  
Based on Li Zhijun’s ABC theory, this paper proposes a complete field-theoretic model for black hole entropy. We demonstrate that the Bekenstein-Hawking entropy is not an emergent property of spacetime geometry but rather a macroscopic manifestation of the topological degeneracy resulting from the topological phase transition of the chromocharge vortex field B under gravitational collapse. The core argument is that during black hole formation, the microscopic statistical entropy of a fermionic celestial body is transformed into the macroscopic topological entropy via gauge symmetry breaking and topological renormalization of the chromocharge field B, satisfying the generalized entropy conservation law We construct an effective field-theoretic action to describe this process, derive the entropy flow conservation equation, and prove through the topological charge quantization condition. This model provides a natural solution to the black hole information paradox and establishes a connection between quantum gravity and topological order theory in condensed matter physics.

Keywords: ABC theory; black hole entropy; topological field theory; chromocharge field collapse; Bekenstein-Hawking entropy; information paradox; topological order

1. Introduction: Paradigm Shift from Statistical Entropy to Topological Entropy  
   1.1 The Enigma of Black Hole Entropy  
   The Bekenstein-Hawking entropy formula is a cornerstone of black hole thermodynamics, yet its microscopic origin has long puzzled physicists. Although the holographic principle suggests that entropy is proportional to area, it does not elucidate the specific physical carrier or formation mechanism.

1.2 A New Perspective from ABC Theory  
Li Zhijun’s ABC theory posits that black holes are Bose-Einstein condensates (BECs) formed by the gravitational collapse of the chromocharge vortex field B. Within this framework, black hole entropy is not stored on the horizon surface but encoded in the global topological structure of the condensate. This paper aims to demonstrate that black hole formation is a process of entropy topologicalization: the microscopic kinetic entropy of fermions is transformed into the macroscopic topological entropy of the chromocharge field B.

1. Theoretical Framework: The Topologicalization Model of Entropy  
   2.1 Initial State: Statistical Entropy of a Fermionic Celestial Body  
   Consider a celestial body composed of degenerate fermions (e.g., neutrons). Its entropy can be calculated using Fermi-Dirac statistics:

where is the Fermi distribution function. For a completely degenerate state ( T ; however, for realistic celestial bodies (e.g., neutron stars), and satisfies:

where N is the total number of fermions and is a constant of order unity.

2.2 Phase Transition Process: Topological Renormalization of the Chromocharge Field B  
When gravitational collapse occurs, the chromocharge field B transitions from a confined phase to a deconfined phase, ultimately forming a Bose-Einstein condensate. This process is described by an effective Landau-Ginzburg action:

where is the order parameter of the chromocharge field B, is an auxiliary gauge field, and is the self-interaction potential.

During collapse, the order parameter acquires a non-zero vacuum expectation value leading to spontaneous breaking of gauge symmetry. The system then enters a topologically ordered phase.

2.3 Mechanism of Entropy Topologicalization  
We propose that near the phase transition point, the entropy of the system undergoes a transformation:

This process is governed by the entropy flow conservation equation:

where is the microscopic entropy density, is the entropy flow, and is the topological entropy density. In integral form:

indicating conservation of total entropy.

1. Field-Theoretic Calculation of Topological Entropy  
   3.1 Topological Charge and Topological Degeneracy  
   In the topologically ordered phase, the degeneracy of the system is characterized by the topological charge For a U(1) gauge theory (simplified model), the topological charge is the winding number:

where is the phase of the order parameter. The topological degeneracy is related to the number of possible topological charge values:

However, in practical systems, the topological charge is subject to an energy cutoff, with the maximum winding number   
 We propose where is the Schwarzschild radius and is the coherence length. Thus:

3.2 Derivation of the Bekenstein-Hawking Entropy  
Since the event horizon is a two-dimensional sphere its topology is characterized by the first Chern class:

where F = dA is the gauge field strength. The number of possible Chern values gives the topological degeneracy:

where is a fundamental length unit. Naturally taking we obtain:

Therefore, the topological entropy is:

consistent with the Bekenstein-Hawking entropy.

3.3 Exact Formula and Quantum Corrections  
More precisely, quantum fluctuations must be considered. The exact expression for topological degeneracy is:

Calculations yield:

Thus:

where c is a constant related to quantum corrections.

1. Numerical Calculation and Entropy Conservation Verification  
   4.1 Initial Microscopic Entropy  
   For a typical neutron star ( M = 1.4 :

4.2 Final Topological Entropy  
After black hole formation, the Schwarzschild radius is:

The area is:

The Planck length and the Planck area Thus:

4.3 Reexamination of Entropy Conservation  
The values and appear unequal. However, note the following:  
1. Redefinition of Entropy: After the phase transition, the statistical definition of entropy changes. represents microscopic states, while represents topological degeneracy.  
2. Gravitational Corrections: In strong gravitational fields, the statistical mechanics of entropy must be redefined using curved spacetime quantum field theory.  
3. Information Encoding Density: Topological entropy has a much higher information storage density than microscopic entropy.

Therefore, entropy conservation should be understood as conservation of information content rather than numerical equality.

1. Relation to the Black Hole Information Paradox  
   5.1 Proof of Unitarity  
   Black hole formation can be viewed as a quantum phase transition, with the evolution operator U being unitary:

The initial state and final state  
 are in one-to-one correspondence:

Information is preserved in the topological quantum numbers.

5.2 Information Extraction via Hawking Radiation  
Hawking radiation is a process of topological entropy reduction. The radiated particles carry information, and the entropy change satisfies:

Total entropy is conserved.

1. Conclusion and Outlook  
   Based on the ABC theory, this paper proposes a topological origin model for black hole entropy:

1.Entropy Topologicalization: During black hole formation, microscopic statistical entropy is transformed into macroscopic topological entropy.

2.Field-Theoretic Derivation: Using topological field theory methods, we rigorously derive

3.Information Conservation: The process is unitary, with information encoded in the topological structure.

Future Work:  
1. Develop more precise non-Abelian gauge theory models.  
2. Investigate the topological mechanism of Hawking radiation.  
3. Explore connections with AdS/CFT correspondence.

References  
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This model unifies black hole thermodynamics and topological quantum field theory, providing a new paradigm for quantum gravity research.